ABSTRACT

In this paper we present SPADES, an interactive graphics based software package for SPAtial mechanism DESign. The program provides a platform for the synthesis of a mechanism that guides a body through either three or four prescribed positions in space.

The purpose of this work is to assemble the current spatial 4C synthesis theory into a software package that is useful for spatial mechanism design and research.

INTRODUCTION

The synthesis of planar mechanisms is inherently a two dimensional problem. Therefore, the design techniques are well suited to a drafting table, blackboard, etc. This is not true of spatial mechanisms. The inherent three dimensionality of these mechanisms makes such two dimensional graphical constructions difficult. For these mechanisms it is useful for the designer to be able to visualize the entire problem in its three dimensions. Modern computer workstations provide the high speed graphics capabilities which make possible real-time visualization of spatial mechanisms. SPADES uses the three dimensional graphics capabilities of a Silicon Graphics Indigo² desktop workstation to provide the interactive environment needed to design spatial 4C mechanisms. Previous efforts which utilized three dimensional graphics in computer-aided mechanism design are summarized in Erdman (1995) and Erdman (1993) and are presented in Rubel and Kaufman (1977), Barris, Kota, Riley, and Erdman (1988), Thatch, and Myklebust (1988), and Larochelle et al (1993).

SPADES is a computer graphics based interactive program for designing spatial mechanisms formed by a closed chain consisting of four cylindric(C) joints. Cylindric joints allow both translation along and rotation about an axis, hence, they are two degree of freedom joints. The result of rigidly connecting four C joints is a two degree of freedom spatial closed chain mechanism, referred to as a spatial 4C mechanism. SPADES facilitates the synthesis of spatial 4C mechanisms to guide a body through three or four finitely separated positions in space.

The theory for the design of spatial mechanisms for four position rigid body guidance is analogous to that for planar mechanisms. In the planar case the designer specifies four positions in the plane and computes the set of points in the moving body which have four positions on a circle, see Hartenberg and Denavit (1964) and Burmester (1888). These points are the moving pivots of planar RR dyads compatible with the four positions and they form a circular cubic curve called the circle point curve. The points that are the centers of these circles are the corresponding fixed pivots of the planar RR dyads and they form a cubic curve called the center point curve. In the case of spatial 4C synthesis the designer specifies four positions in space. The set of lines in the moving body which maintain a constant distance (i.e. normal distance and twist) from a fixed line form a congruence of lines called the moving line congruence. The points that are the centers of these circles are the corresponding fixed pivots of the planar RR dyads and they form a cubic curve called the center point curve. In the case of spatial 4C synthesis the designer specifies four positions in space. The set of lines in the moving body which maintain a constant distance (i.e. normal distance and twist) from a fixed line form a congruence of lines called the moving line congruence. The set of corresponding fixed lines of the CC dyads form the fixed line congruence, see Larochelle (1995) and Bottema and Roth (1979) for further dis-
cussions about these line congruences and their properties. The major difference between planar and spatial finite position synthesis is the essential three dimensionality of spatial mechanisms. While synthesis curves for planar mechanisms may be sketched or plotted in two dimensions the synthesis congruences for spatial mechanisms and the mechanisms themselves must be viewed in their full three dimensional form. The designer requires the ability to manipulate the synthesis congruences and view them in an arbitrary orientation in order to gain an understanding of the available choices of fixed and moving axes. Furthermore, once the linkage has been specified the evaluation of its motion also requires the ability to view the linkage in a three dimensional environment.

The remainder of the paper proceeds as follows. First, we review the underlying synthesis theory utilized by SPACES to perform synthesis for three and four position rigid body guidance. This is followed by a summary of the kinematic analysis tools employed by SPACES and a discussion regarding the organizational structure of the software. Finally, we conclude with a design case study which illustrates the utility of SPACES as a research and design tool and discuss the planned future development of SPACES.

THREE POSITION SYNTHESIS

The methodology utilized for performing the dimensional synthesis of spatial 4C mechanisms for three position rigid body guidance involves synthesizing two CC dyads separately and then joining them with a coupler to form a complete closed chain mechanism. This algorithm is based upon the works of Larochelle (1994), Suh and Radcliffe (1978), and Tsai and Roth (1973).

We consider one CC dyad of the spatial 4C mechanism as shown in Fig. 1. Let the axis of the fixed joint be specified by the dual vector \( \hat{u} \) measured in the fixed reference frame and let the moving axis be specified by \( \hat{\lambda} \) measured in the moving frame \( M \), see Bottema and Roth (1979) for further discussion of dual vectors.

In a CC dyad there is a link which connects the moving line and the fixed line. This link is assumed to be rigid and therefore the twist and normal distance between the fixed and moving lines of the dyad remain constant. The constant twist condition may be expressed as,

\[
\mathbf{u} \cdot [A] \hat{\lambda} - \cos \alpha = 0 \tag{1}
\]

and the constant moment condition as,

\[
\mathbf{u} \cdot ([A] \hat{\lambda}^0 + [D][A] \hat{\lambda} + \mathbf{u}^0 \cdot [A] \hat{\lambda} + a \sin \alpha = 0 \tag{2}
\]

where \([A]\) is the 3 × 3 rotation matrix which describes the orientation of the moving body with respect to the fixed frame and \([D]\) is the 3 × 3 skew-symmetric matrix from the translation vector \( \mathbf{d} \) which locates the origin of the moving frame. Note that Eq. 1 and Eq. 2 are the implicit constraint equations for a spatial CC dyad.

For three position synthesis we first select the moving axis \( \hat{\lambda} \). Second, we write Eq. 1 for each of the desired positions, \( ([A], \mathbf{d})_i, i = 1, 2, 3 \). Finally, we subtract the first equation from the remaining two to arrive at a linear system of equations,

\[
[P][\mathbf{u}] = \mathbf{b} \tag{3}
\]

where,

\[
[P] = \begin{bmatrix}
(l_2 - l_1)^T \\
(l_3 - l_1)^T \\
0 & 0 & 1
\end{bmatrix}
\]

\[
\mathbf{b} = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

\( l_1 \), are the coordinates of the moving pivot in the \( i^{th} \) position, and \( \mathbf{u} \) is a vector in the direction of the desired fixed pivot. We now solve the system of equations and normalize the solution to obtain the vector \( \mathbf{u} \). Next, write Eq. 2 for each of the desired positions, \( ([A], \mathbf{d})_i, i = 1, 2, 3 \). Again, we subtract the first equation from the remaining two to arrive at a linear system of equations,

\[
[P][\mathbf{u}]^0 = \mathbf{b} \tag{4}
\]

where,

\[
[P] = \begin{bmatrix}
(l_2 - l_1)^T \\
(l_3 - l_1)^T \\
\mathbf{u}^T
\end{bmatrix}
\]

\[
\mathbf{b} = \begin{bmatrix}
-(l_1^0 - l_1^0) \cdot \mathbf{u} \\
-(l_2^0 - l_2^0) \cdot \mathbf{u} \\
0
\end{bmatrix}
\]

The desired fixed axis is,

\[
\hat{u} = \begin{bmatrix}
\mathbf{u} \\
\mathbf{u}^0
\end{bmatrix} \tag{5}
\]

Note that the fixed axis \( \hat{u} \) which corresponds to the desired \( \hat{\lambda} \) is unique and that we must perform this procedure for both desired moving axes in order to complete the 4C mechanism synthesis.
FOUR POSITION SYNTHESIS

The design congruences for four position synthesis of spatial 4C closed chain mechanisms are called the fixed line and moving line congruences. They are used in the same way as the planar linkage design curves; the center point and circle point curves. The fixed line congruence is the set of lines that will serve as fixed axes and the moving line congruence is the set of corresponding moving axes. The numerical procedure we use to generate these congruences is presented in detail in Larochelle (1995) and we briefly summarize the process here.

The generalization of Burmester's planar four position theory to four spatial displacements leads us to consider the complementary screw quadrilateral $S_{12}S_{23}S_{34}$, where $S_{12}$, $S_{13}$, $S_{14}$, $S_{23}$, $S_{24}$, and $S_{34}$ are the six relative screw axes associated with the four prescribed spatial positions of a moving body, see Fig. 2 and Roth (1967a, 1967b) and Bottema and Roth (1979). We use the lines which define the complementary screw quadrilateral to define a spatial 4C mechanism and identify the quadrilateral as the home configuration of the parameterizing 4C mechanism.

The screw axis of the displacement of the coupler of the parameterizing 4C mechanism from its home configuration to any other valid assembly is a fixed axis compatible with the given four spatial positions and hence is a line of the fixed line congruence. Murray and McCarthy (1994) show that solving the spatial triangle associated with the two lines which define the input crank in its home configuration, Eq. 6 and Eq. 7, results in the screw axis of the displacement of the coupler of the parameterizing 4C mechanism. The dual vector equation of this spatial triangle (shown in Fig. 3) may be written as,

\[ \sin \frac{\Delta \hat{\theta}}{2} = \sin \frac{\Delta \hat{\phi}}{2} \cos \frac{s_{12}}{2} \]
\[ \cos \frac{\Delta \hat{\phi}}{2} = \cos \frac{s_{12}}{2} \cos \frac{\Delta \hat{\phi}}{2} \]
\[ \sin \frac{\Delta \hat{\phi}}{2} \sin \frac{\Delta \hat{\phi}}{2} \times s_{12} \cdot s_{23} \]  

\[ \hat{\theta} = \theta - \hat{\theta}_0, \Delta \hat{\phi} = \phi - \hat{\phi}_0, \hat{\theta}_0 \] is the relative dual angle between the input crank and the fixed link in the home configuration, and $\hat{\phi}$ is the relative dual angle between the coupler and the input crank in the home configuration. By solving the spatial triangle we obtain a fixed axis compatible with the given four spatial positions which is parameterized by the input angle of the parameterizing 4C linkage.

\[ \hat{\theta} = \lambda + \epsilon \lambda^0 \]

\[ \hat{u} = u + c \hat{u}^0 \]

Figure 1. A SPATIAL CC DYAD

Figure 2. THE PARAMETERIZING SPATIAL 4C MECHANISM

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KINEMATIC ANALYSIS

Here we introduce the notation for the link lengths and joint angles. We then present the equations which are utilized to perform a kinematic analysis of the spatial 4C mechanism. These equations are derived by utilizing the matrix method of Hartenberg and Denavit (1964). Finally, we present the linkage classification method employed by $S_9$ades.

Position Analysis

The link lengths and joint angles are as defined in Tbl. 1. The coupler angle $\phi$ as a function of the input angle $\theta$ is,

$$\phi(\theta) = \arctan\left(\frac{B}{A}\right) \pm \arccos\left(\frac{C}{A^2 + B^2}\right) \tag{8}$$

where

$$A = \sin(\eta) \sin(\gamma) \cos(\alpha) \cos(\theta) - \sin(\alpha) \sin(\eta) \cos(\gamma)$$
$$B = -\sin(\eta) \sin(\gamma) \sin(\theta)$$
$$C = \cos(\beta) - \cos(\eta) \sin(\alpha) \sin(\gamma) \cos(\theta) - \cos(\alpha) \cos(\eta) \cos(\gamma).$$

The output angle $\psi$ as a function of the input angle $\theta$ and the coupler angle $\phi$ is,

$$\psi(\theta, \phi) = \arctan\left(\frac{B}{A}\right) \tag{9}$$

where,

$$A = \frac{1}{-\sin(\beta)} \left\{ \cos(\eta)(\cos(\alpha) \sin(\gamma) - \cos(\gamma) \cos(\phi) \sin(\alpha)) + \sin(\eta) \cos(\phi) \sin(\gamma) + \sin(\eta) \cos(\phi) \sin(\theta) \right\}$$
$$B = \frac{1}{\sin(\beta)} \left\{ \cos(\eta) \sin(\alpha) \sin(\theta) + \sin(\eta) \cos(\phi) \sin(\theta) + \sin(\eta) \cos(\alpha) \cos(\phi) \sin(\theta) \right\}.$$

The output coupler angle $\delta$, i.e. the angle between the coupler and the driven crank is,

$$\delta(\theta, \psi) = \arctan\left(\frac{B}{A}\right) \tag{10}$$

where,

$$A = \frac{1}{\sin(\eta)} \left\{ \cos(\alpha)(\cos(\gamma) \sin(\beta) + \cos(\beta) \sin(\gamma) \cos(\psi)) - \right.$$
The driven coupler translation $c_1$ (i.e. the translation along the driving axis) is given by,

$$c_1(\theta, \phi, \gamma, d_1) = \frac{A}{B}$$  \hspace{1cm} (11)

where,

\[
A = d_1 \sin(\gamma) \sin(\psi) + a \cos(\theta) \cos(\psi) + a \cos(\gamma) \sin(\theta) \sin(\psi) + h \cos(\delta) - b - g \cos(\psi) \\
B = \sin(\eta) \sin(\delta).
\]

The driven coupler translation $c_2$ (i.e. the translation along the driven moving axis) is given by,

$$c_2(\theta, \phi, \psi, c_1) = \frac{A}{B}$$  \hspace{1cm} (12)

where,

\[
A = h \cos(\phi) \cos(\theta) + c_1 \sin(\alpha) \sin(\theta) + a \cos(\theta) - h \cos(\alpha) \sin(\phi) \sin(\theta) - g - b \cos(\psi) \\
B = \sin(\beta) \sin(\psi).
\]

Finally, the translation along the driven fixed line $d_2$ is given by,

$$d_2(\theta, \phi, \psi, c_1, c_2) = \frac{A}{B}$$  \hspace{1cm} (13)

where,

\[
A = h \cos(\phi) \sin(\theta) - c_1 \cos(\theta) \sin(\phi) + a \sin(\theta) + h \cos(\alpha) \cos(\theta) \sin(\phi) - b \cos(\gamma) \sin(\psi) + c_2 \cos(\beta) \sin(\gamma) + c_2 \cos(\gamma) \cos(\psi) \sin(\beta) \\
B = - \sin(\gamma).
\]

**Mechanism Classification**

Associated with each spatial 4C mechanism is a spherical image. The spherical image is a spherical four-bar mechanism with link lengths equal to the angular twist of the links of the 4C mechanism, see Duffy (1980). Pades classifies spatial 4C mechanisms according to the linkage type of the corresponding spherical image. The linkage type of the spherical image is computed using the method presented in Murray and McCarthy (1995).
to satisfy given design specifications. The program is organized such that each phase of the design process is carried out in a separate graphics window. Corresponding to each window is a button listed on the right hand side of the screen in the available menus column labeled with the name of the window. To select a window to work in use the mouse to push the appropriate button in the available menus column. The menu associated with that graphics window is then displayed on the far right hand side of the screen, see Fig. 4.

At start-up there are two windows displayed; the linkage window and the linkage info window. The menu associated with the linkage window is displayed on the far right side of the screen. There is no menu associated with the linkage info window. To begin the design process buttons in the right menu are pressed. These buttons include Pos Syn, Pos Data, Movie, and Mode. As the buttons are pressed the required windows and their associated buttons and menus are created.

Position Synthesis

To design a linkage to guide a body through three or four positions in space press the pos syn button in the linkage menu. The pos syn window will be created as well as the pos syn menu. Four positions are represented as coordinate frames in space. The position frames have their axes color coded; \( x = \textcolor{red}{\text{red}} \), \( y = \textcolor{green}{\text{green}} \), and \( z = \textcolor{blue}{\text{blue}} \). The orientation of the position frames is specified by their longitude, latitude, and roll angles, see Larochelle and McCarthy (1995). Position orientations are specified by graphically moving a frame (select the desired position number in the pos syn menu) to the desired orientation or by typing the desired longitude, latitude, and roll angles directly into the linkage info window. The location of the origin of the position is specified numerically in the linkage info window.

Once the four positions are specified the corresponding design congruences are generated by pressing either the congos, guide map, or points button in the pos syn menu. Alternatively, three position synthesis is accomplished by pressing the 3 pos syn button.
**Design Congruences.** After the four precision positions are specified and the congruences created, the selection of the cranks of the mechanism is accomplished in the congos, guide map, or points window. By choosing the appropriate button in the congos menu both the driven crank and the driving crank are selected from the congruences. Picking a crank involves selecting a desired line from the design congruences. First, press either the *pk driving* or *pk driven* buttons in the congos menu. Then, place the mouse pointer on the desired line in the congos window where planes of the moving line congruence are yellow and planes of the fixed line congruence are red. Press the left or right mouse button to select a line. Both lines of the crank will then be displayed in the congos window. The driving crank lines are green while the driven crank lines are red. Note that the same color coding of the lines is used in the linkage window. After both cranks have been chosen the *analyze* button in the congos menu is used to create the corresponding linkage which is then displayed in the linkage window. Note that in the congos window each plane of the congruence is selected by a generator line. The ability to select a line from a plane other than its generator is enabled by selecting *use generators* in the preferences menu. This will allow you to select a line from a plane that passes through, or nearest, a desired point.

**Guide Map.** After the four precision positions are specified the selection of the cranks of the mechanism can alternatively be accomplished in the guide map window. By choosing the appropriate button in the pos syn menu a guide map is generated. The guide map is a two-dimensional visual representation of all of the mechanisms generated by the congruences. In the guide map all possible selections of lines from the congruences are coded according to mechanism type. Pressing *col key* in the guide map menu will generate a mechanism type color legend. By selecting a point on the guide map with the mouse both the driven crank and the driving crank are determined and the mechanism is analyzed and displayed in the linkage window. The note above regarding *use generators* in the preferences menu also applies to the guide map.

**Points.** After the four precision positions are specified the selection of the cranks of the mechanism can also be accomplished in the points window. By choosing the appropriate button in the pos syn menu the points window is generated. In this window you can select a mechanism from the congruences by specifying a point that you want an axis of the mechanism to pass through. Note that the distance from the selected line of the congruence nearest the desired point is displayed and that the linkage is automatically analyzed and displayed in the linkage window.

**Three Position Synthesis.** After the precision positions are specified, synthesis of a mechanism that guides a moving body through the first three of the four positions can be accomplished in the 3 pos syn window. By choosing the appropriate button in the pos syn menu the 3 pos syn window is generated. In this window you can specify either the fixed or moving line of each crank and the corresponding moving or fixed line will be computed and the linkage will automatically be analyzed and displayed in the linkage window.

**Motion Simulation**

Once synthesized, the planned motion of the 4C mechanism can be viewed by pressing the *movie* button in the linkage menu. SPADERS will plan a path for the mechanism and will actuate both degrees of freedom such that the moving body will pass through the four, or three, desired positions. The speed of the movie can be altered by selecting *adj. movie speed* in the preferences menu. Press the *movie* button again to terminate the movie function and return to normal SPADERS operation. Note that pressing *pos data* in the linkage menu will display the driving crank rotation angle and translation distance in each of the four positions.

**Pull-Down Menus**

At the top of the main SPADERS display are standard windows pull-down menus which open and close files that contain saved designs as well as three additional menus: *Preferences, Rot. Anim.*, and, *Trans. Anim.* The *Preferences* menu presents various options which are made available to the user throughout the design process to enhance the capabilities of SPADERS:

- **Linkage Colors**- Toggles between a steel grey mechanism and one which has a green driving crank and a red driven crank.
- **Show Axes**- Toggles display of the fixed frame axes in the linkage window.
- **Show Pos-n-Linkage**- Toggles display of the four desired positions in the linkage window.
- **Use Generators**- Toggles display of a dialogue window to prompt the user to select a specific line in a plane of the congruence rather than use the line which was used to generate that plane.
- **Adj. Movie Speed**- Toggles display of a dialogue window to prompt the user for a new movie display speed.

The *Rot. Anim.* and *Trans. Anim.* menus present options for manually actuating the two degrees of freedom at the driving fixed axis. The video cassette-player type selections available for each degree of freedom are: *Play, FastForward, ReversePlay, Slow — Motion, Super — Slow, and Stop.*
DESIGN CASE STUDY

In this section we present an example of the design of a spatial 4C mechanism for four position rigid body guidance. The goal is to move a pallet off of a flexible assembly line into a convenient position to perform an assembly operation on the underside of the pallet and then to return the pallet to the assembly line. A moving coordinate frame was assigned to the pallet and the 4C mechanism is to be attached to the pallet at the points; \([-1 \ 0.25 \ 0]^T\) and \([1 \ 0.25 \ 0]^T\). It is at these two points that holes are to be drilled in the pallet. These holes will serve as journal bearings for the moving C joints of the 4C mechanism. This application was suggested by Mark Senti and his associates at GSMA Systems Inc, Melbourne, FL and was first addressed in Larochelle (1995). The desired positions are listed in Tbl. 2. The fixed and moving congruences associated with this set of positions are shown in Fig. 5. From the computed congruences we seek a 4C mechanism with a driving crank which has a moving line that passes through the point \(p_{dvg} = [-1 \ 0.25 \ 0]^T\) and a driven crank with a moving line that passes through the point \(p_{dyn} = [1 \ 0.25 \ 0]^T\); both points are given with respect to the moving frame. The synthesis will be accomplished by utilizing the the points synthesis feature of \(S\)ADES which was previously discussed, see Fig. 4 for a snapshot of the points feature in use.

For each plane of the moving congruence the distance \(h\) from the plane to the point \(p_{dvg}\) was computed. The line of the moving congruence nearest the desired point \(p_{dvg}\) was found to be at a distance of \(h_{min} = 0.032148\). We obtain the driven dyad in an analogous manner. The line of the moving congruence nearest the point \(p_{dyn}\) was found to be at a distance of \(h_{min} = 0.019717\). \(S\)ADES automatically computes the fixed axes which correspond to these moving axes and displays the mechanism and the pertinent data in the design environment. Finally, we show the resulting non-Grashof spatial 4C mechanism in Fig. 6 and list its link lengths in Tbl. 3. Moreover, the dual input dual for the mechanism with the moving body in each of the four prescribed positions are found in Tbl. 2. In Fig. 6 the mechanism is shown with: a green driving crank, a red driven crank, and the moving body in position 4 and the lines which it to the coupler.

FUTURE DEVELOPMENT

\(S\)ADES is written in the C programming language and utilizes the GL graphics library which is available on Silicon Graphics computers. In the near future we plan to port \(S\)ADES to the new OpenGL graphics library. This will allow \(S\)ADES to be compiled and run on any platform which has a C compiler and the OpenGL libraries.

Future synthesis tools planned for \(S\)ADES include a path planning module to plan the coordinated movement of the two degrees of freedom to guide the moving body in some optimal fashion. Possible optimization objective functions may include: minimizing the translation required along the driving fixed line, minimizing the number of required algebraic sign changes for the driving force and driving torque, minimizing the maximum required forces and torques, minimizing the required power, etc. Moreover, we plan to build upon the work of Ling and Hu (1997) and develop a module for sizing the links and locating their connections to avoid collisions during actuation of the mechanism.

CONCLUSIONS

In this article we have presented \(S\)ADES, an interactive software package for designing spatial 4C closed chains. Incorporated into \(S\)ADES are modules for performing synthesis for three and four position rigid body guidance. Moreover, \(S\)ADES provides the designers with three dimensional motion planning and animation of the mechanism.

It is our hope that this research and design tool will facilitate the design and manufacture of mechanisms to solve spatial motion problems. The source code for \(S\)ADES and the \(S\)ADES User’s Guide are available, free of charge, from the author.

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Table 2. THE 4 PRESCRIBED POSITIONS

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<td>FIXED</td>
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Table 3. THE DESIRED 4C MECHANISM
ACKNOWLEDGMENTS

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